choice of method to implement in Chapter 5, are well argued from the analysis.

The style of the book is clear and readable, and the misprints few. Interconnections between the methods treated are pointed out, and the text is well integrated.

All in all, this is a delightful volume.

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3 [7.75].—YUDELL L. LUKE, *Mathematical Functions and their Approximations*, Academic Press, New York, 1975, xvii + 568 pp., 23.5cm. Price \$14.50.

One of the best selling mathematics books of all time (discounting text books) is *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables* edited by M. Abramowitz and I. Stegun, National Bureau of Standards Applied Mathematics Series 55, U. S. Government Printing Office, Washington, D. C., 1964 (or AMS 55 for short). To quote from the preface of the book of Luke under review:

"The cutoff date for much of the material in AMS 55 is about 1960. In the past 15 years much valuable new information on the special functions has appeared. In some quarters, it has been suggested that a new AMS 55 should be produced. This is not presently feasible. The task would be gigantic and would consume much time. Most certainly the economics of the situation forbids such a program. A feasible approach is a handbook in the spirit of AMS 55, which, in the main, supplements the data given there.

The present volume can be conceived as an updated supplement to that portion of AMS 55 dealing with mathematical functions."

This extensive quote is given so that the reader will have some idea what the author purports to do. Actually, this is not an adequate summary of Luke's book, and the title is misleading. The book is about hypergeometric functions, some of the generalized hypergeometric functions and special cases. Hypergeometric functions are very important, and a number of books need to be written about them from various points of view. But they are not the only useful mathematical functions, so the title of this book is unfortunate.

However, the content of a book is much more important than the title. The main focus is the problem of computing the hypergeometric series

$$ {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; x) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{x^n}{n!}, $$

where the shifted factorial $(a)_n$ is defined by $(a)_n = a(a+1)\cdots(a+n-1)$, $n = 1, 2, \ldots$, $(a)_0 = 1$. Among the special cases treated in some detail are the binomial function $(1+z)^a = {}_1F_0(-a; -; -z)$, $\ln(1+z) = z\,{}_2F_1(1, 1; 2; -z)$, $\exp(z) = {}_0F_0(-; -; z)$, $\arcsin z = z\,{}_2F_1(1/2, 1/2; 3/2; z^2)$, $\arctan z = z\,{}_2F_1(1/2, 1; 3/2; -z^2)$, the incomplete gamma function $\gamma(\nu, z) = \nu^{-1}z^{\nu}\,{}_1F_1(\nu, \nu+1; -z)$, and the error function $\mathrm{Erf}(z) = z\,{}_1F_1(1/2; 3/2; -z^2)$. The Bessel function $J_\nu(z) = [(z/2)^\nu/\Gamma(\nu+1)]\,{}_0F_1(-; \nu+1; -z^2/4)$, the confluent hypergeometric function ${}_1F_1(a, b; c; z)$, and the classical hypergeometric function ${}_2F_1(a, b; c; z)$ are each treated in separate chapters.

The longest chapter is "The generalized hypergeometric function ${}_pF_q$ and the G-function." The function ${}_3F_2(a, b, c; d, e: 1)$ is very important, and it was completely omitted from AMS 55. For example, the functions

$$ {}_3F_2(-n, n+\alpha+\beta+1, -x; \alpha+1, -N; 1) $$

are a set of polynomials orthogonal with respect to

$$\binom{X+\alpha}{x}\binom{N-x+\beta}{N-x}\bigg/\binom{N+\alpha+\beta+1}{N}$$

for $n, x = 0, 1, \ldots, N$, when $\alpha, \beta > -1$ or $\alpha, \beta < -N$. This important set of orthogonal polynomials, usually called Hahn polynomials but actually discovered in 1875 by Chebyshev, is starting to play an increasingly important role in applied mathematics. Karlin and McGregor came across them in population genetics; Ph. Delsarte used them in coding theory; and Cooper, Hoare and Rahman used them to solve a probability problem which arose in statistical mechanics. In addition, they play an important role in coupling of angular momenta in quantum mechanics. Unfortunately, with the exception of a paragraph on page 168, there is no mention of applications of ${}_3F_2$'s. The orthogonality is never mentioned. Delsarte's work is less than five years old and the work of Cooper, Hoare and Rahman was not written when this book was published, so the author has a good reason for not including these specific references. However, the work of Karlin and McGregor is over fifteen years old and is relatively well known. I know at least ten other papers on these polynomials. So some reference to them should have been given somewhere in this book. The same goes for the other discrete orthogonal polynomials of Charlier, Krawtchouk, and Meixner. The treatment of them in AMS 55 is so sketchy as to be essentially worthless. A supplement to AMS 55 in this area would have been very useful. There are interesting questions concerning the location of the zeros of some of these polynomials which fit in perfectly with the philosophy stated in the preface: "Numerical values of functions are but a facet of the overall problem. We desire approximations to compute functions and their zeros, to simplify mathematical expressions such as integrals and transforms, and to facilitate directly the mathematical solution of a wide variety of functional equations such as differential equations, integral equations, etc."

The main portions of this book deal with two different methods of approximating hypergeometric functions. In the first, the function is expanded in a series of Chebyshev polynomials and the partial sums of this series are used to approximate the function. A Chebyshev polynomial expansion of a function on $(0, a)$ is nothing more than a Fourier expansion of this function treated as an even function on $(-a, a)$, so it is somewhat surprising not to find Zygmund's book, *Trigonometric Series,* in the bibliography. Zygmund does not consider the problem of actually adding the terms of a series to get a number, but it has so many useful bits of information that anyone who uses Chebyshev expansions should be familiar with the mathematics in Zygmund's book. Some of the tricks which can be used to numerically evaluate a series are given in Luke's book, such as Clenshaw's observation that Horner's method of nesting a power series can be used on series whose terms satisfy a recurrence relation.

The other method used to compute hypergeometric functions is to construct Padé approximations, i.e. rational functions which are best local approximations at a given point among all rational functions of given degree. In the chapter on Bessel functions the very useful idea of using recurrence relations in a backward way to construct minimal solutions is treated.

There are a few misprints, mostly of a trivial nature. However, I was unable to figure out what (48) on page 170 should be. The comment on page 166 after (14) is only correct if the series terminate. Formula (24) on page 167 is not due to Rice (1944). It is a limiting case of a very important formula of John Dougall and was stated explicitly in the 1920's if not in Dougall's 1907 paper. Unfortunately, Dougall's sum of a "two-balanced, very well poised ${}_7F_6$" is not mentioned, nor is Whipple's transformation of a balanced ${}_4F_3$ to a very well poised ${}_7F_6$. These are the formulas which lie behind many of the explicit sums that are known.

The remark on page 243 is correct, and it would have been helpful if some indication of its importance had been given. It is the reason for the existence of Hahn polynomials as orthogonal polynomials, and it has been used to prove deep inequalities for some ${}_3F_2$'s. As it stands, it looks like just another random comment, while it is actually a very important remark.

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4 [14].—PETER HENRICI, *Applied and Computational Complex Analysis*, Vol. 1, John Wiley & Sons, Inc., New York, 1974, xv + 682 pp., 24cm. Price $24.95.

The importance of complex function theory in applied mathematics both from a qualitative and from a numerical standpoint is indisputable. Thus, the appearance of a modern treatment emphasizing the manipulative and computational aspects of the subject will be welcomed by those interested in applying mathematics in many areas, and by numerical mathematicians in particular.

This volume, the first of three, is devoted to power series, analytic continuation, complex integration, elementary conformal mapping, polynomials, and partial fractions. The second, scheduled to appear shortly, will include material pertinent to the analysis of ordinary differential equations, special functions, integral transforms, and continued fractions, while the last volume will treat topics bearing on the study of partial differential equations.

The treatment of power series is divided into two chapters. The first, on formal power series, discusses the formal manipulation of series in considerably more detail than usual, including composition and reversion as well as formal differentiation and algebraic operations. Convergence is not introduced until the second chapter in which, anticipating the need for analytic functions of matrices in discussing systems of differential equations, the variable is taken as an element of a general Banach algebra. Most of the standard results on convergence, analyticity, composition and inversion carry through with this generalization, and so do many of the properties of the elementary transcendental functions.

A distinctive feature of the next chapter, on analytic continuation, is the discussion of the techniques necessary to make the Weierstrass process a constructive one by reducing the number of terms included in the successive power series expansions.

The fourth chapter, of over 100 pages, is primarily devoted to complex integration and its many applications, but also includes an analytic treatment of the Laurent series, and of the principle of the argument.

The discussion of conformal mapping in Chapter 5 (to be extended in the next volume) begins with an exposition of the geometric approach to complex analysis. A thorough treatment of the Moebius transformation is followed by a brief development of the theory of holomorphic functions and their equivalence to analytic functions. Applications of the techniques developed so far to problems of two-dimensional electrostatics, fluid dynamics and elasticity are given before presenting the general mapping theorem, the symmetry principle, and the Schwarz-Christoffel mapping function.

Although there are interesting variations and extensions, and a refreshing selection of new illustrative examples, these first chapters are primarily standard material. The next two are less usual. Chapter 6, on polynomials, begins with material such as the